1. Match the following terms to the diagram.

Given the rectangular prism with face BCFE as one of its bases. Use each value ONLY ONCE.

- C 1. Edge
- E 2. Lateral Face
- A 3. Base
- D 4. Vertex
- B 5. Height

- A. Rectangle ADGH
- B. $\overline{HF}$
- C. $\overline{AD}$
- D. Point B
- E. Rectangle HDCF

2. After looking at the rectangular prism to the right, a young lady in the class raises her hand and says, "Could I use rectangle ADCB as my base instead of rectangle BHGC?" Yes either could be considered a base because they both have parallel plane congruent opposite faces.

3. Properly name the following prisms.

a) Name: CUBE  

b) Name: TRIANGULAR PRISM  

c) Name: OBLIQUE RECT. PRISM  

or PARALLELOGRAM PRISM  

d) Name: HEXAGONAL PRISM  

e) Name: TRIANGULAR PRISM  

f) Name: RECTANGULAR PRISM  

g) Name: TRIANGULAR PRISM  

h) Name: OCTAGONAL PRISM  

4. Mike doesn't understand how volume works for a prism and Henry is trying to explain it to him. "It's what is inside the shape... for example, if you calculated the area of one piece of paper and then stacked 100 pieces of paper on top of each other it would create a prism and the volume would be the area of the one piece of paper multiplied by the height of the stack." MIKE IS STILL CONFUSED, can you give another example to explain stacking CD cases.

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- AREA OF 1 CD CASE
- HEIGHT OF STACK
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5. Cavalieri’s principle says that these two prisms have equal volume. Explain why that is true?

* SAME HEIGHT
* SAME CROSS SECTIONS
* AT ALL LEVELS (12 cm²)

6. If the volume of the cube is \((4)(4)(4) = 64\) cm², what is the volume of the oblique prism if it has been tilted at 60°?

* NO CHANGE IN VOLUME *
  - SAME HEIGHT
  - SAME BASE
  - AREA CROSS SECTIONS

7. Jenny says that the two prisms DO NOT have the same volume because the cross sections are not the same. Renee disagrees; she says that it isn’t the shape that has to be the same it is the area. Renee thinks they have the same volume. Who is right and why?

Renee IS CORRECT. WHILE THE SHAPES ARE DIFFERENT THE AREA OF THE CROSS SECTION IS EQUAL!!

\[
\frac{1}{2}(5)(12) = 30 \text{ cm}^2 \quad 5(6) = 30 \text{ cm}^3 \quad \text{VOLUME =}
\]

8. An enclosed glass box contains 1620 in³ of water. When the glass box is tilted on its side the water shifts places. What is the relationship of the water before and after the tilting? What is the height of the water when the box is tiled upright?

\[
(5)(27)(12) = 1620 \text{ in}^3
\]

a) Volume is simply moved.

b) \(12(18)(x) = 1620 \text{ in}^3\)

\(x = 7.5 \text{ in.}\)
9. Randy says “Cylinder volume is easy – it is done the same way as a prism except its base is a circle.” What does Randy mean by this?

\[ \text{Prism} \quad V = \text{base area} \times \text{height} \]

\[ \text{Cylinder} \quad V = \pi r^2 h \]

10. Ryllie looks at the two cylinders below and says that the oblique cylinder has less volume because it is at an angle. The volume for the right cylinder is \( V = \pi (3)^2 (h) \) and the volume for the oblique cylinder is \( V = \pi (3)^2 (\sin 73^\circ)(h) \) because it is at an angle. Is she correct? NOPE!

\[ \text{Heights are=} \quad \bullet \text{Bases} = \text{Area} \]

\[ \text{The height was not changed by making the cylinder oblique.} \]

11. Jared wants to test out a new theory..... Instead of having the cross area sections the same as Cavalieri suggested he wants to half the radius of one cross section and then double the height to make up for it. He believes because he divided the radius by 2 but doubled the height that the volumes should be equal. Is he correct? Explain.

\[ \text{Short/wide cylinder} \]

\[ V = Bh \]

\[ = \pi (4)^2 (5) \]

\[ = 80\pi \text{ cm}^2 \]

* This didn’t work because \( r \) is squared in the relationship.

\[ \text{Tall/narrow cylinder} \]

\[ V = Bh \]

\[ = \pi (2)^2 (10) \]

\[ = 40\pi \text{ cm}^2 \]

12. A rectangular prism and a cylinder have the same height and the same volume. What is the length of the side of the prism's square base?

\[ V = \pi (4)^2 h \]

\[ V = x^2 h \]

\[ \pi (16) h = x^2 h \]

\[ 16\pi = x^2 \]

\[ \sqrt{16\pi} = x \]

\[ 4\sqrt{\pi} = x \]
1. The same rectangular prism is provided three times below but in each instance a DIFFERENT BASE has been highlighted. Calculate the volume for each but change the base dimensions.

\[ V = Bh \]
\[ V = (10 \times 2)(11) = 220 \text{ cm}^2 \]
\[ V = (11 \times 2)(10) = 220 \text{ cm}^2 \]
\[ V = (10 \times 11)(2) = 220 \text{ cm}^2 \]

What do you notice about the volumes of these three examples? Why didn’t changing the base change the volume? All the same. In a rectangular prism there are multiple faces that could be bases.

2. Determine the volume of the prisms. (Lines that appear perpendicular are perpendicular.)

\[ V = Bh \]
\[ V = \left(\frac{1}{2}(7+14)(6)\right)(8) \]
Volume = 504 cm³

d)
\[ V = Bh \]
\[ V = \left(\frac{1}{2}(3)(8)\right)(11) \]
Volume = 115.5 cm³

e)
\[ V = Bh \]
\[ V = (5)(5)(11) \]
Volume = 275 cm³

f)
\[ V = Bh \]
\[ V = \left(\frac{1}{2}(4)(12)\right)(14) \]
Volume = 336 cm³

g)
\[ V = Bh \]
\[ V = (8)(6)(7) \]
Volume = 336 cm³

h)
\[ V = Bh \]
\[ V = \left(\frac{1}{2}(8+14)(5)\right)(11) \]
Volume = 605 cm³

i)
\[ V = Bh \]
\[ V = (8 \times 8)(8) \]
Volume = 512 cm³
3. Determine the volume of the prism.

a) Equilateral Triangular Prism

\[ V = Bh \]
\[ = \left( \frac{1}{2} (6) (3\sqrt{3}) \right) (10) \]

Volume = \( 90\sqrt{3} \text{ cm}^3 \) (E)

b) Rectangular Prism

\[ x^2 + 5^2 = 13^2 \]
\[ x = 12 \]
\[ V = Bh \]
\[ = \left( \frac{1}{2} (12)(5) \right) (15) \]

Volume = \( 450 \text{ cm}^3 \)

c) Regular Hexagonal Prism

\[ V = Bh \]
\[ = \left( \frac{1}{2} (2\sqrt{3})(24) \right) (9) \]

Volume = \( 216\sqrt{3} \text{ cm}^3 \) (E)

d) Diagonal

\[ \text{diagonal} = 3\sqrt{2} \text{ cm} \]
\[ V = Bh \]
\[ = (3\times3\sqrt{3})(15) \]

Volume = \( 135 \text{ cm}^3 \)

e) Oblique Prism

\[ V = Bh \]
\[ = (10 \times 5\sqrt{3})(8) \]

Volume = \( 240\sqrt{3} \text{ cm}^3 \) (E)

f) Regular Hexagonal Prism

\[ V = Bh \]
\[ = \left( \frac{1}{2} (2\sqrt{3})(24) \right) (7) \]

Volume = \( 168\sqrt{3} \text{ cm}^3 \) (E)

g) Right Triangle

\[ V = Bh \]
\[ = \left( \frac{1}{2} (4)(2\pi) \right) (8) \]

Volume = \( 32\sqrt{3} \text{ cm}^3 \) (E)

h) Right Rectangular Prism

\[ \tan 70^\circ = \frac{8}{h} \]
\[ h = 2.91176 \]
\[ V = Bh \]
\[ = (8 \times 2.91176)(10) \]

Volume = \( 232.94 \text{ cm}^3 \) (E)

i) Trapezoidal Prism

\[ V = Bh \]
\[ = \left( \frac{1}{2} (6)(4) \right) (7) \]

Volume = \( 84 \text{ cm}^3 \)
1. Determine the volume of the following prisms.
(Lines that appear to be perpendicular are perpendicular and lines that appear to be parallel are.)

a) \[ V = Bh = (70 + 9)(12) \]
   \[ V = 948 \text{ cm}^3 \]

b) \[ V = \text{Volume of box} - \text{Volume of cut-out} \]
   \[ V = (12)(18)(10) - (5)(8)(10) \]
   \[ V = 1760 \text{ cm}^3 \]

c) \[ V = Bh = (70 + 12)(8) \]
   \[ V = 656 \text{ cm}^3 \]

d) \[ V = Bh = (20 + 48)(10) \]
   \[ V = 680 \text{ cm}^3 \]

e) \[ V = Bh = (48 + 6 + 15 + 48)5 \]
   \[ V = (117)(5) \]
   \[ V = 585 \text{ cm}^3 \]

f) \[ V = Bh = (75 + 45)(6) \]
   \[ V = 720 \text{ cm}^3 \]
e) \[ V = Bh = (22 \times 18 + \frac{1}{2}(6)(18))(20) = (396 + 54)(20) \]

Volume = \(9000 \text{ cm}^3\)

f) \[ V = Bh = (\frac{1}{2}(10+21)(8)) - (2)(8)(8) = (124 - 4)(8) \]

Volume = \(960 \text{ cm}^3\)

g) \[ V = Bh = (\frac{1}{2}(5)(5\sqrt{3}))(8) \]

Volume = \(100\sqrt{3} \text{ cm}^3\)

h) \[ \begin{align*} x^2 + 2^2 &= 5^2 \\ x &= 4.5825 \end{align*} \]

Volume = \(494.92 \text{ cm}^3\)

(2 dec.)

i) \[ \begin{align*} x^2 + 5^2 &= 13^2 \\ x &= 12 \end{align*} \]

\[ V = Bh = (40 + 60)(6) \]

Volume = \(600 \text{ cm}^3\)

j) \[ V = Bh = (4\sqrt{2})^3 \]

Volume = \(432\sqrt{2} \text{ cm}^3\)

(E)
1. Determine the volume of the cylinder.

a) \[ V = \pi r^2 h = \pi (4)^2 (7) \]
Volume = \( 112\pi \text{ cm}^3 \) (E)

b) \[ V = \pi r^2 h = \pi (3)^2 (8) \]
Volume = \( 72\pi \text{ cm}^3 \) (E)

c) \[ V = \pi r^2 h = \pi (5)^2 (9) \]
Volume = \( 225\pi \text{ cm}^3 \) (E)

d) \[ V = \pi r^2 h = \pi (5)^2 (23) \]
Volume = \( 575\pi \text{ cm}^3 \) (E)

e) \[ V = \pi r^2 h = \pi (3)^2 (10) \]
Volume = \( 250\pi \text{ cm}^3 \) (E)

2. Determine the volume of the cylinder.

a) \[ V = \pi (8)^2 (12) - \pi (4)^2 (12) \]
Volume = \( 576\pi \text{ cm}^3 \) (E)

b) \[ V = \pi (2.5)^2 (8) - \pi (2)^2 (8) \]
Volume = \( 18\pi \text{ cm}^3 \) (E)

c) \[ V = \pi (4)^2 (5) - \pi (1)^2 (5) \]
Volume = \( 75\pi \text{ cm}^3 \) (E)
3. Determine the volume of the solid.

a) \[ V = \frac{1}{4} \pi (6)^2 (10) \]
Volume = $90 \pi \text{ cm}^3$ (E)

b) \[ V = \frac{3}{4} \pi (2)^2 (5) \]
Volume = $15 \pi \text{ cm}^3$ (E)

c) \[ V = Bh = \left(12 + \frac{\pi (3)^2}{2}\right)(10) \]
\[ = (12 + 4.5\pi)(10) \]
Volume = $120 + 45\pi \text{ cm}^3$ (E)

d) \[ V = Bh = \left(\frac{\pi (3)^2}{2} + 24 + \frac{6(4)}{2}\right)(12) \]
\[ = (4.5\pi + 24 + 12)(12) \]
\[ = 54\pi + 432 \]
Volume = $54\pi + 432 \text{ cm}^3$ (E)

e) \[ V = Bh = \frac{\pi (4)^2}{2}(12) \]
\[ = 8\pi (12) \]
Volume = $96 \pi \text{ cm}^3$ (E)

f) \[ V = Bh = \left(2\pi(6)^2 + \frac{\pi (9)^2}{2} \cdot \frac{(30)^2}{2}\right) \]
\[ = (2016 + 45\pi)(15) \]
\[ = (264 + 36\pi)(15) \]
Volume = $3960 + 540\pi \text{ cm}^3$ (E)

g) Diameter of hole = 2 cm

h) Square hole with sides of 3 cm

\[ V = Bh = (4(4) - \pi (1)^2)(9) \]
\[ = (16 - \pi)(4) \]
Volume = $64 - 4\pi \text{ cm}^3$ (E)

i) Two circular holes, diameter of holes is 6 cm.

\[ V = Bh = (\pi (6)^2 - \pi (9)^2)(13) \]
\[ = (36\pi - 81\pi)(13) \]
\[ = (46\pi)(13) \]
Volume = $598\pi \text{ cm}^3$ (E)
G.GMD.3 WORKSHEET #4 – PATTERSON

1. Match the following terms to the diagram.

Given the square pyramid.

- **D** 1. Slant Height
- **F** 2. Apex
- **E** 3. Height
- **B** 4. Lateral Edge
- **A** 5. Face
- **C** 6. Vertex

2. Jeff missed class and Dillon is explaining the notes. “The slant height and the height of the pyramid basically mean the same thing.” Is this summary of height correct? Explain. **THEY ARE DIFFERENT!!**
   - Slant Height (s) is the height of a face
   - Height (h) is perpendicular distance from apex to base face

3. Properly name the pyramid.

   a) Name: ____________________________
   b) Name: ____________________________
   c) Name: ____________________________
   d) Name: ____________________________

   **HEXAgonAL PyRAMID** **SQUARE PyRAMID** **RECTANGULAR PyRAMID** **TRIANGULAR PyRAMID**

4. Two pyramids with the same base are side by side. One is a right pyramid and the other is an oblique pyramid. If the oblique pyramid has been tilted to an angle of 80º, what is volume relationship between the two pyramids?

   **RIGHT** **OBLIQUE**
   
   \[
   \text{BASE AREA} = \text{BASE AREA} \\
   \text{HEIGHT} = \text{HEIGHT} \\
   \frac{1}{3} Bh = \frac{1}{3} Bh \\
   \text{EQUAL VOLUME}
   \]**
5. Determine the volume of the pyramid.

a) Square Pyramid

\[ V = \frac{1}{3} Bh \]
\[ = \frac{1}{3} (6)(6)(14) \]

Volume = \[168 \text{ cm}^3\]

b) Rectangular Pyramid

\[ h^2 + 4^2 = 10^2 \]
\[ h = \sqrt{10^2 - 4^2} = 9.16515 \]

\[ V = \frac{1}{3} (8)(6)(9.16515) \]

Volume = \[146.64 \text{ cm}^3\] (2 dec.)

c) Regular Hexagonal Pyramid

\[ V = \frac{1}{3} Bh \]
\[ = \frac{1}{3} \left( \frac{1}{2} ap \right)h \]
\[ = \frac{1}{6} \left( 4\sqrt{3} \right)(48)(20) \]

Volume = \[640\sqrt{3} \text{ cm}^3\] (E)

d) Square Pyramid

\[ V = \frac{1}{3} Bh \]
\[ = \frac{1}{3} (10)(10)(\sqrt{119}) \]
\[ h = \sqrt{119} \]
\[ h = 10.90871 \]

Volume = \[363.62 \text{ cm}^3\] (E)

e) Equilateral Triangular Pyramid

\[ l^2 + s^2 = 13^2 \]
\[ l = 12 \text{ cm} \]
\[ h^2 + s^2 = 12^2 \]
\[ h = \sqrt{119} \]
\[ h = 10.90871 \]

\[ A = \frac{1}{3} Bh \]
\[ = \frac{1}{3} \left( \frac{1}{2} (12)(6\sqrt{3}) \right)(10) \]

Volume = \[120\sqrt{3} \text{ cm}^3\] (E)

f) Square Pyramid

\[ V = \frac{1}{3} Bh \]
\[ = \frac{1}{3} (10)(10)(5\sqrt{3}) \]

Volume = \[166\frac{2}{3}\sqrt{3} \text{ cm}^3\] (E)
6. Determine the volume of the pyramid.

a) Square Pyramid

\[ 17^2 = 8^2 + h^2 \]

\[ h = \sqrt{17^2 - 8^2} = 15 \text{ cm} \]

\[ V = \frac{1}{3} Bh = \frac{1}{3} (60 \times 16) (15) = 1280 \text{ cm}^3 \]

Volume = \( 1280 \text{ cm}^3 \)

b) Triangular Pyramid

\[ V = \frac{1}{3} Bh = \frac{1}{3} \left( \frac{1}{2} \times 8 \times 9 \right) (8) = \left( \frac{1}{3} \times 36 \times 8 \right) = 96 \text{ cm}^3 \]

Volume = \( 36 \text{ cm}^3 \) (2 dec.)

c) Frustum of a Pyramid

\[ h^2 + 6^2 = 10^2 \]

\[ h = 8 \text{ cm} \]

\[ V = Bh + \frac{1}{3} Bh = (12)(8)(5) + \frac{1}{3} (12)(12)(8) = 240 + 384 = 624 \text{ cm}^3 \]

Volume = \( 1104 \text{ cm}^3 \)

d) Tetrahedron

\[ V = \frac{1}{3} Bh = \frac{1}{3} \left( \frac{1}{2} \times 4 \times \sqrt{3} \right) (12) = \left( \frac{1}{3} \times 2 \times \sqrt{3} \times 4 \right) = 8 \sqrt{3} \text{ cm}^3 \]

Volume = \( 96 \sqrt{3} \text{ cm}^3 \) (approx.)

e) Trapezoidal Prism

\[ V = \frac{1}{3} Bh = \frac{1}{3} \left( \frac{1}{2} \times 5 \times 14 \right) (5) = \left( \frac{1}{3} \times \frac{5}{2} \times 5 \times 14 \right) = 224 \text{ cm}^3 \]

Volume = \( 20 \text{ cm}^3 \)

f) Triangular Pyramid

\[ V = \frac{1}{3} Bh = \frac{1}{3} \left( \frac{1}{2} \times 8 \times 4 \right) (4) = \left( \frac{1}{3} \times 8 \times 4 \times 4 \right) = 272 \sqrt{3} \text{ cm}^3 \]

Volume = \( 272 \sqrt{3} \text{ cm}^3 \) (approx.)
1. Determine the volume of the cone.

a) \[ V = \frac{1}{3} \pi (3)^2 (8) \]
Volume = \( 24 \pi \text{ cm}^3 \) (E)

b) \[ V = \frac{1}{3} \pi (2)^2 (9) \]
Volume = \( 12 \pi \text{ cm}^3 \) (E)

c) \[ V = \frac{1}{3} \pi (4)^2 (12) \]
Volume = \( 64 \pi \text{ cm}^3 \) (E)

d) \[ V = \frac{1}{3} \pi (9)^2 (10) \]
Volume = \( 270 \pi \text{ cm}^3 \) (E)

e) \[ V = \frac{1}{3} \pi (5)^2 (6) \]
Volume = \( 15 \pi \text{ cm}^3 \) (E)

f) \[ r^2 + 15^2 = 17^2 \]
\[ r = 8 \]
\[ V = \frac{1}{3} \pi (8)^2 (15) \]
\[ V = 320 \pi \text{ cm}^3 \]

2. Determine the volume of the cone.

a) \[ h^2 + 5^2 = 13^2 \]
\[ h = 12 \text{ cm} \]
\[ V = \frac{1}{3} \pi (5)^2 (12) \]
Volume = \( 100 \pi \text{ cm}^3 \) (E)

b) \[ V = \frac{1}{3} \pi (6)^2 (6\sqrt{3}) \]
Volume = \( 72 \sqrt{3} \pi \text{ cm}^3 \) (E)

c) \[ V = \frac{1}{3} \pi (6)^2 (12) \]
Volume = \( 144 \pi \text{ cm}^3 \) (E)
d) \[ V = Bh + \frac{1}{3}Bh \]
\[ = \pi(3)^2(8) + \frac{1}{3}\pi(3)^2(9) \]
\[ = 72\pi + 27\pi \]
\[ = 99\pi \text{ cm}^3 \]

Volume = \(99\pi \text{ cm}^3\) (E)

e) \[ V = \left(\frac{1}{3}Bh\right)2 + Bh \]
\[ = \left(\frac{1}{3}\pi(4)^2(10)\right)2 + \pi(4)^2(6) \]
\[ = \frac{160\pi}{3} + 96\pi \]
\[ = \frac{320\pi}{3} + 96\pi \]
\[ = \frac{320\pi}{3} + \frac{288\pi}{3} \]
\[ = \frac{608\pi}{3} \]

Volume = \(\frac{202}{3}\pi \text{ cm}^3\) (E)

b) \[ V = \frac{1}{3}Bh \]
\[ = \frac{1}{3}\pi(4)^2(10.9899) \]

\[ \tan 20^\circ = \frac{4}{h} \]
\[ h = 10.9899 \text{ cm} \]

Volume = \(184.14 \text{ cm}^3\) (2 dec.)

g) \[ V = \frac{1}{3}Bh \]
\[ = \frac{1}{3}\pi(6\sqrt{3})^2(9) \]
\[ = \frac{1}{3}\pi(108)(9) \]
\[ = 324\pi \text{ cm}^3 \]

Volume = \(324\pi \text{ cm}^3\) (E)
1. Determine the volume of the solid.

\[ V = \frac{4}{3} \pi r^3 \]

\[ V = \frac{4}{3} \pi (6)^3 \]
\[ \text{Volume} = 288 \pi \text{ cm}^3 \] (E)

\[ V = \frac{4}{3} \pi r^3 \]
\[ V = \frac{4}{3} \pi (3)^3 \]
\[ \text{Volume} = 36 \pi \text{ cm}^3 \] (E)

\[ V = \frac{2}{3} \pi (10)^3 \]
\[ \text{Volume} = \frac{666}{3} \pi \text{ cm}^3 \] (E)

d) Two tennis balls fit exactly in the 48 cm tall cylindrical can. What is the volume of air in the can?

\[ V = Bh - 2 \left( \frac{4}{3} \pi r^3 \right) \]
\[ = \pi (12)^2 (48) - 2 \left( \frac{4}{3} \pi (12)^3 \right) \]
\[ = 6912 \pi - 4608 \pi \]
\[ \text{Volume} = 2304 \pi \text{ cm}^3 \] (E)

f) Surface Area of a sphere = 4\(\pi r^2\). If the surface area of a sphere is 144\(\pi\), then what is its volume?

\[ 144 \pi = 4 \pi r^2 \]
\[ 36 = r^2 \]
\[ 6 = r \]
\[ V = \frac{4}{3} \pi (6)^3 = 288 \pi \text{ cm}^3 \]

g) Surface Area of a sphere = 4\(\pi r^2\). If the surface area of a sphere is 16\(\pi\), then what is its volume?

\[ 16 \pi = 4 \pi r^2 \]
\[ 4 = r^2 \]
\[ r = 2 \]
\[ V = \frac{4}{3} \pi (2)^3 = 10\frac{2}{3} \pi \text{ cm}^3 \]